Benha University
Faculty Of Engineering at Shoubra

# ECE 122
# Electrical Circuits (2)(2017/2018)
# Lecture (03)
# Parallel Resonance

Prepared By :
Dr. Moataz Elsherbini
motaz.ali@feng.bu.edu.eg

# Remember

# Series Resonance

[1] $\omega_o = \frac{1}{\sqrt{LC}}$ , $f_o = \frac{1}{2\pi\sqrt{LC}}$

[2] $Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$ → total series Res.

$Z = R$ (at resonance).

[3] $BW = \omega_2 - \omega_1 = \frac{R}{L} = \frac{\omega_o}{Q}$

[4] $\omega_{1\,actual} = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \cong \omega_o - B/2$ → from half power

$\omega_{2\,actual} = \frac{+R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \cong \omega_o + B/2$ → from half power

} at $Z = \sqrt{2}R$

[5] $\omega_o = \sqrt{\omega_1 \omega_2}$

[6] $Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$

# Parallel Resonance Circuit

It is usually called tank circuit

Ideal Circuits

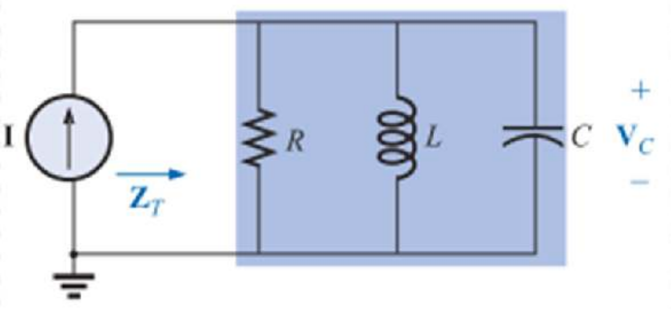



**FIG. 20.21**
*Ideal parallel resonant network.*

Practical Circuits

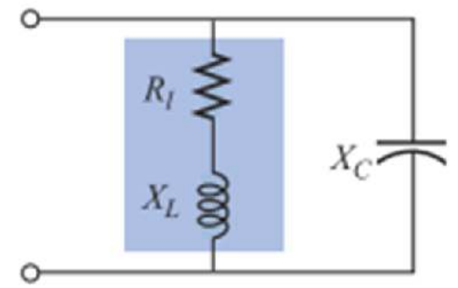
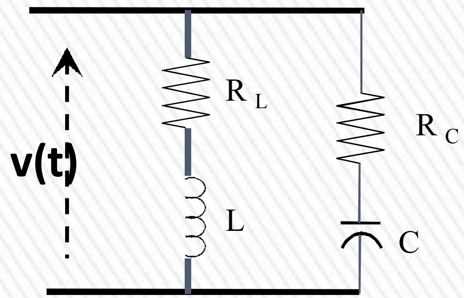



**FIG. 20.22**
*Practical parallel L-C network.*

Complex Configuration



# Ideal Parallel Resonance Circuit

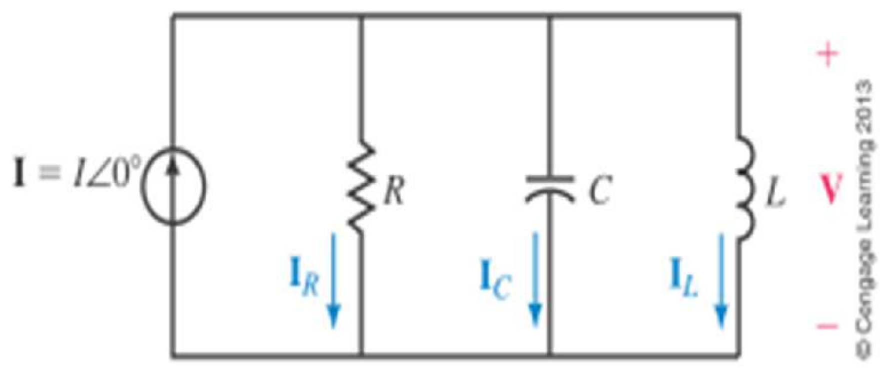
## The total admittance

$$Y = Y_1 + Y_2 + Y_3$$

$$Y = \frac{1}{R} + \frac{1}{(j\omega.L)} + \frac{1}{(-j/\omega.C)}$$

$$Y = \frac{1}{R} + \frac{-j}{\omega L} + j\omega C$$

$$Y = \frac{1}{R} + j(\omega C - 1/\omega L)$$




## Condition for Ideal Parallel Resonance

Resonance occurs when the imaginary part of **Y** is zero

$$\omega C - \frac{1}{\omega L} = 0$$

$$X_C = X_L.$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

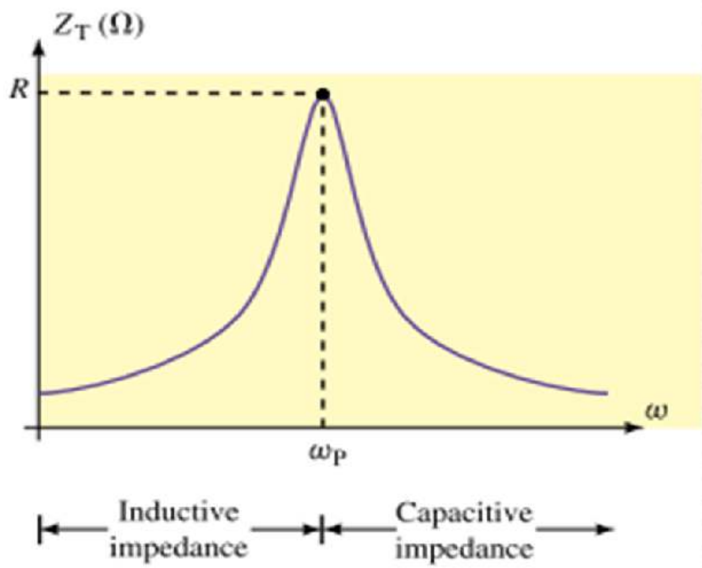
# Ideal Parallel Resonance Circuit

## At parallel resonance:

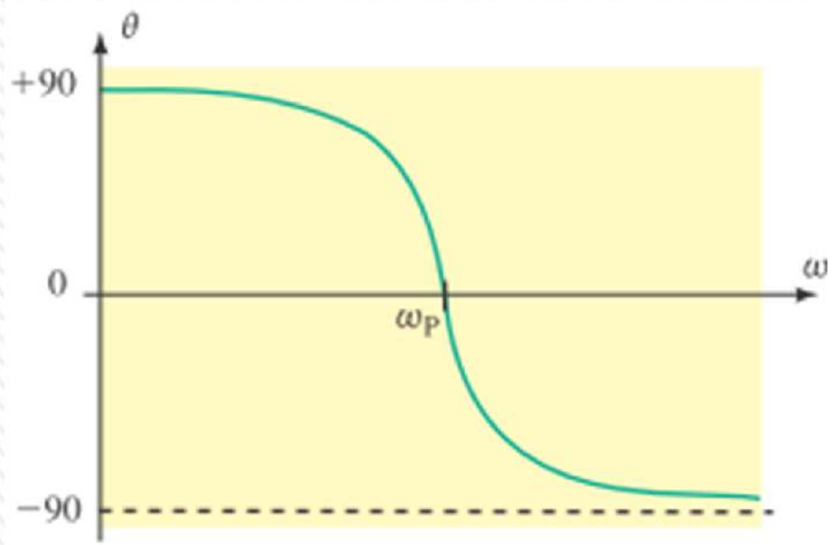
- ✓ At resonance, the admittance consists only conductance G = 1/R.
- ✓ The value of current will be minimum since the total admittance is minimum.
- ✓ The voltage and current are in phase (Power factor is unity).
- ✓ The inductor reactance and capacitor reactance canceled, resulting in a circuit voltage simply determined by Ohm’s law as:

$$\mathbf{V} = \mathbf{I}R = IR\angle 0^\circ$$

- ✓ The frequency response of the impedance of the parallel circuit is shown




exactly opposite to that in series resonant circuits,



# Ideal Parallel Resonance Circuit

The Q of the parallel circuit is determined from the definition as

$$Q_P = \frac{\text{reactive power}}{\text{average power}} = \frac{V^2/X_L}{V^2/R}$$

$$Q_P = \frac{R}{X_{LP}} = \frac{R}{X_C}$$

**Reciprocal of series case**

The current

$$\mathbf{I}_R = \frac{\mathbf{V}}{\mathbf{R}} = \mathbf{I}$$

$$\mathbf{I}_L = \frac{\mathbf{V}}{X_L\angle 90^\circ} = \frac{V}{R/Q_P}\angle -90^\circ = Q_P I\angle -90^\circ$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{X_C\angle -90^\circ} = \frac{V}{R/Q_P}\angle 90^\circ = Q_P I\angle 90^\circ$$

- ✓ The currents through the inductor and the capacitor have the same magnitudes but are 180 out of phase.
- ✓ Notice that the magnitude of current in the reactive elements at resonance is Q times greater than the applied source current.

# Ideal Parallel Resonance Circuit

- **Parallel resonant circuit has same parameters as the series resonant circuit.**

Resonance frequency:

$$\omega_p = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

Half-power frequencies:

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \quad \text{(rad/s)}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \quad \text{(rad/s)}$$

Bandwidth and Q-factor:

$$\text{BW} = \omega_2 - \omega_1 = \frac{1}{RC} \quad \text{(rad/s)}$$

$$\text{BW} = \frac{\omega_P}{Q_P} \quad \text{(rad/s)}$$

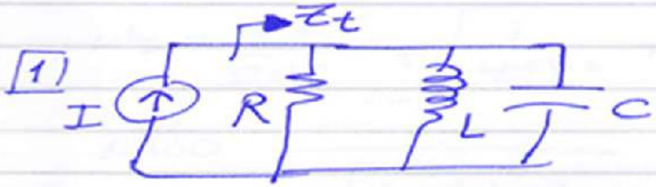
$$\text{BW} = \frac{\omega_P}{R(\omega_P C)} = \frac{X_C}{R}\omega_P$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

# ملخص قوانين المحاضرة

## Ideal circuit

[1]




[2] $X_L = X_C$ at Resonance

∴ $\omega_p = \frac{1}{\sqrt{LC}}$ rad/s

[3] $Y = \frac{1}{Z} = Y_1 + Y_2 + Y_3$

$= \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$

[4] at Resonance $G = \frac{1}{R}$ (G admittance)

& V, I inphase.



[5] $Q_p = \frac{R}{X_C} = \frac{R}{X_L} = \frac{\omega_p}{BW}$

$= R/\omega L = \omega RC$

[6] $BW = \omega_2 - \omega_1 = \frac{1}{RC}$ rad/s

$= \omega_p / Q_p$

[7] $\omega_1 = \frac{-1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$

$\omega_2 = \frac{+1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$ rad/s

Note for Midband

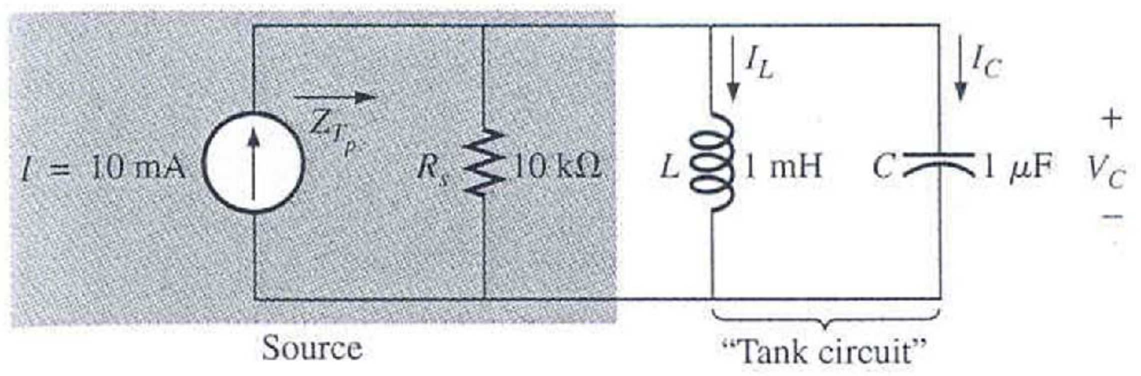
$\omega_1 = \omega_p - B/2$

$\omega_2 = \omega_p + B/2$

# Example

# Example (1)

## Consider the circuit shown in Figure




**Given the parallel resonant circuit of Fig.16 composed of ideal elements:**

a) Determine the resonant frequency fp

b) Find the total impedance at resonance

c) Calculate the quality factor and bandwidth of the system

d) Find the voltage VC at resonance

e) Determine current IL and IC

f) Calculate the cut-off frequencies f1 and f2

# Example (1)

*Solution*

a)
$$f_p = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1x10^{-3})(1x10^{-6})}}$$
$$=5.03 \text{ kHz}$$

b)
$$Z_T = R_s \parallel Z_L \parallel Z_C = R_s = 10k\Omega$$

c)

$$Q_p = \frac{R_s}{X_L} = \frac{R_s}{2\pi f_p L} = \frac{10x10^3}{2\pi(5.03x10^3)(1x10^{-3})}$$
$$=316.41$$

$$BW = \frac{f_p}{Q_p} = \frac{5.03x10^3}{316.41} = 15.9Hz$$

# Example (1)

d) $V_C = IZ_T = (10x10^{-3})(10x10^{3}) = 100\ V$

e)

$$I_L = \frac{V_L}{X_L} = \frac{V_C}{2\pi f_p L} = \frac{100}{2\pi(5.03x10^3)(1x10^{-3})} = \frac{100}{31.6} =$$

$$I_C = \frac{V_C}{X_C} = \frac{100}{31.6} = 3.16$$

f)

$$f_2 = \frac{f_p}{2Q_p} + f_p\sqrt{(\frac{1}{2Q_p})^2 + 1}$$

$$= \frac{5030}{2x316.4} + 5030\sqrt{(\frac{1}{2x316.4})^2 + 1} = 5041.9Hz$$

$$f_1 = -\frac{f_p}{2Q_p} + f_p\sqrt{(\frac{1}{2Q_p})^2 + 1}$$

$$= 5026.02Hz$$

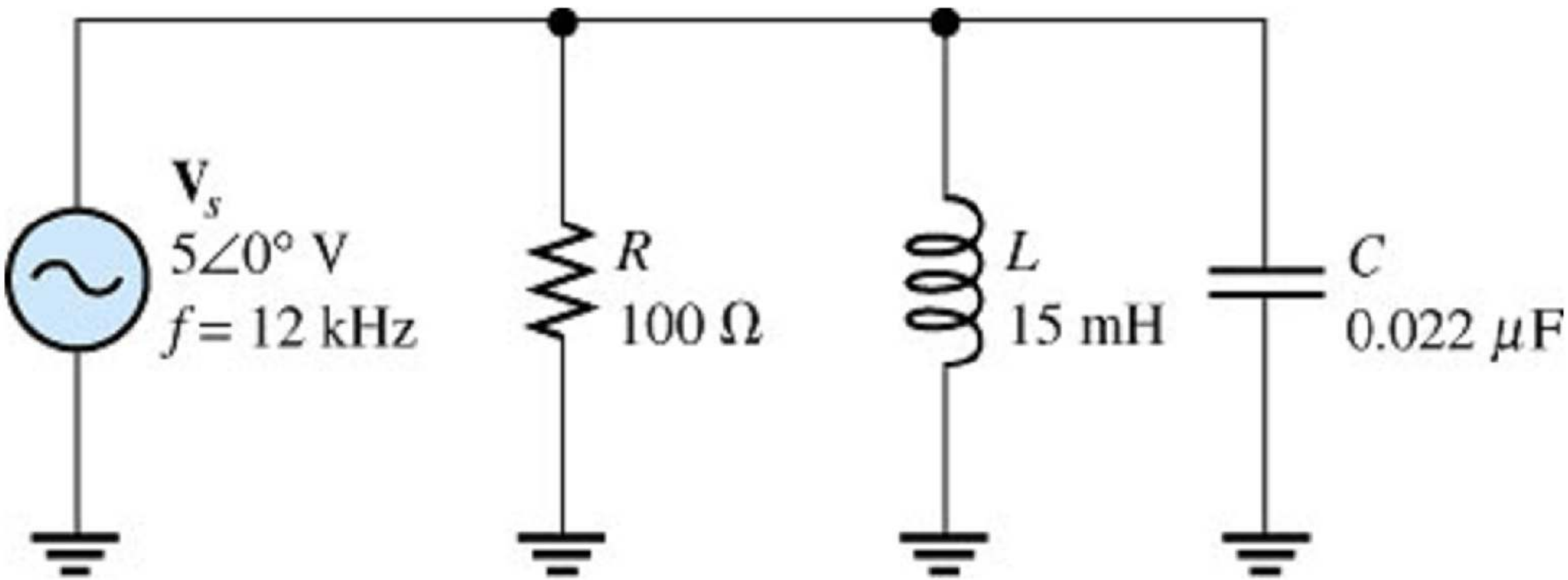
# Example (2)

Find the following parameters:

a). Circuit Impedance in polar form.

b). Is the circuit more inductive or more capacitive?

c). At what frequency does the circuit change its reactive characteristic from inductive to capacitive or vice-versa?



# Example (2)

a). Circuit Impedance in polar form.

$$X_L = 2\pi fL = 2\pi(12\text{ kHz})(15\text{ mH}) = 1131\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(12\text{ kHz})(0.022\ \mu\text{F})} = 603\ \Omega$$

$$\mathbf{Z} = \frac{1}{\dfrac{1}{100\angle 0°\ \Omega} + \dfrac{1}{1131\angle 90°\ \Omega} + \dfrac{1}{603\angle -90°\ \Omega}}$$

$$= \frac{1}{10\text{ mS} - j0.884\text{ mS} + j1.66\text{ mS}} = \mathbf{99.7\angle{-4.43}°\ \Omega}$$

$$Y_T = 10\text{mS} + j0.776\text{mS} = 0.01003\text{S} \angle 4.43°$$

$$Z_T = 1/Y_T = 1/(.01003\text{S}\angle 4.43°) = 99.7\text{ohms}\angle -4.43°$$

# Example (2)

b). Since θ of $Z_T$= -4.43°, the circuit is slightly capacitive.

c).

$$X_L < X_C$$

$$2\pi f L < \frac{1}{2\pi f C}$$

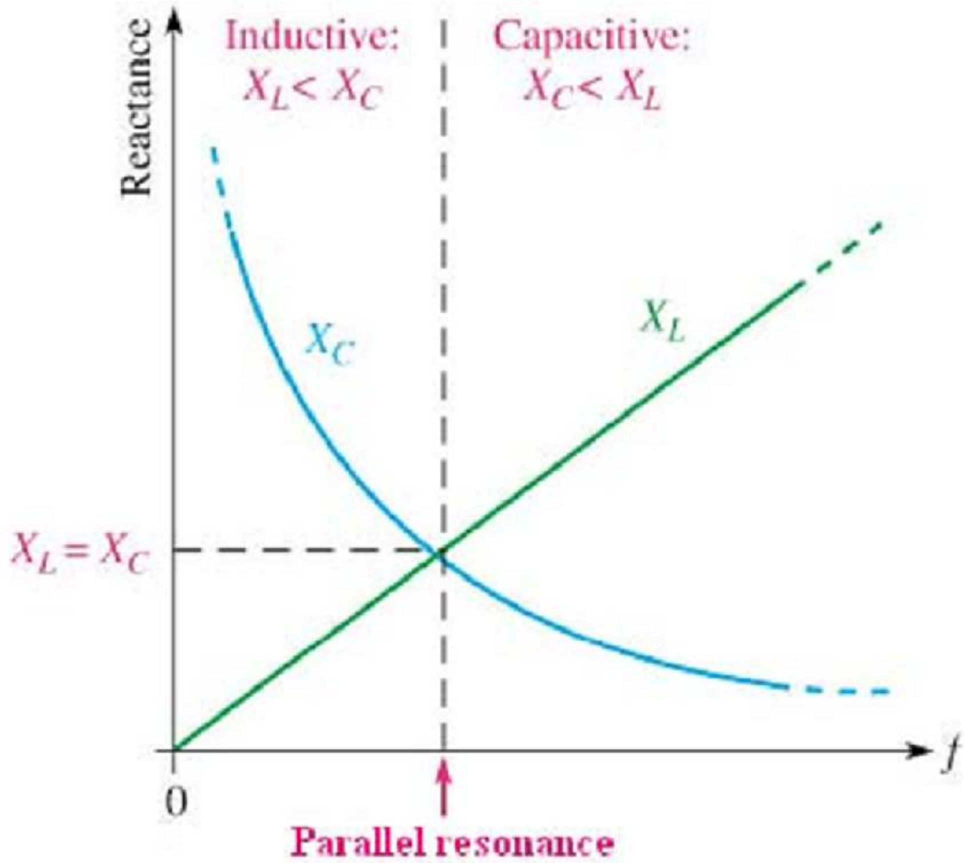
$$f^2 < \frac{1}{4\pi^2 LC}$$

$$f < \frac{1}{\sqrt{4\pi^2 LC}}$$

$$f < \frac{1}{2\pi\sqrt{LC}}$$

$$f < \frac{1}{2\pi\sqrt{(15\,\text{mH})(0.022\,\mu\text{F})}}$$

$$f < \mathbf{8.76\ kHz}$$

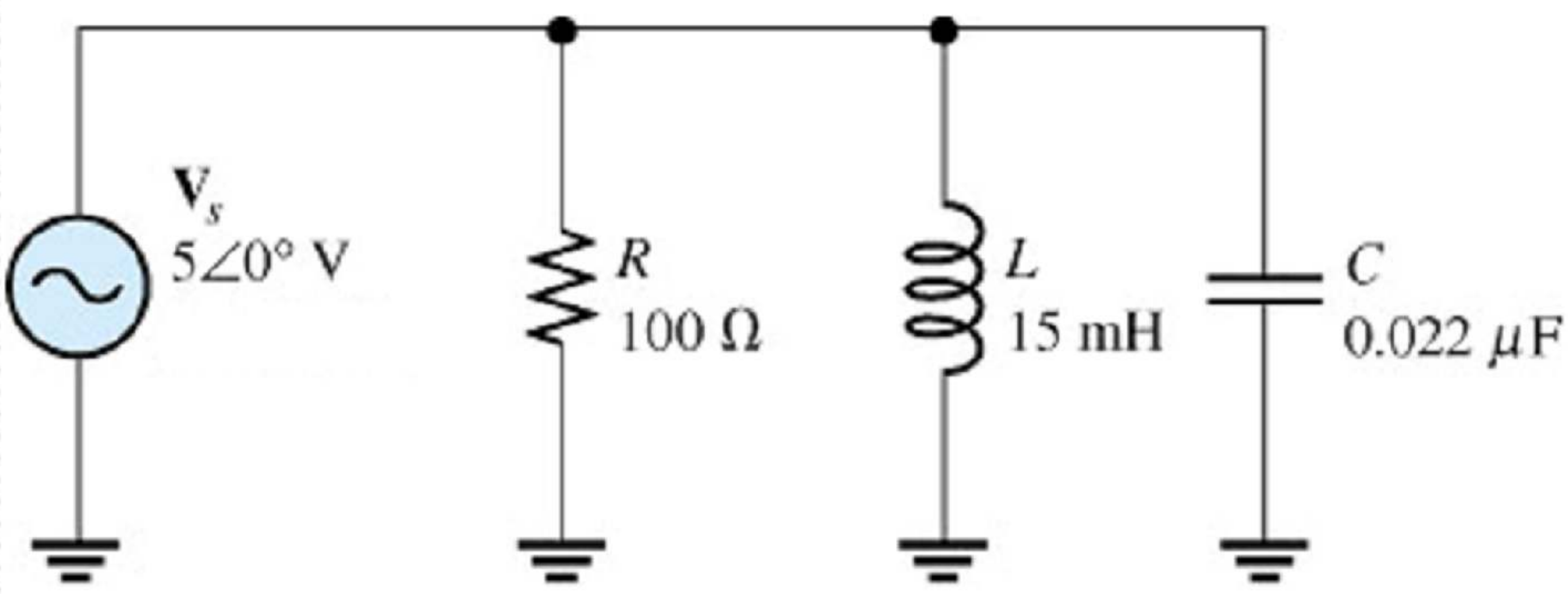


## Example (3)

Find the following parameters:

a). All currents and voltages in polar form if the source frequency is 12KHz.

b). The current phasor diagram.



# Example (3)

a). All currents and voltages in polar form.

$$I_T = \frac{V_s}{Z} = \frac{5\angle 0^\circ \text{ V}}{99.7\angle -4.43^\circ \ \Omega} = 50.2\angle 4.43^\circ \text{ mA}$$

$$I_R = \frac{V_s}{R} = \frac{5\angle 0^\circ \text{ V}}{100\angle 0^\circ \ \Omega} = 50\angle 0^\circ \text{ mA}$$

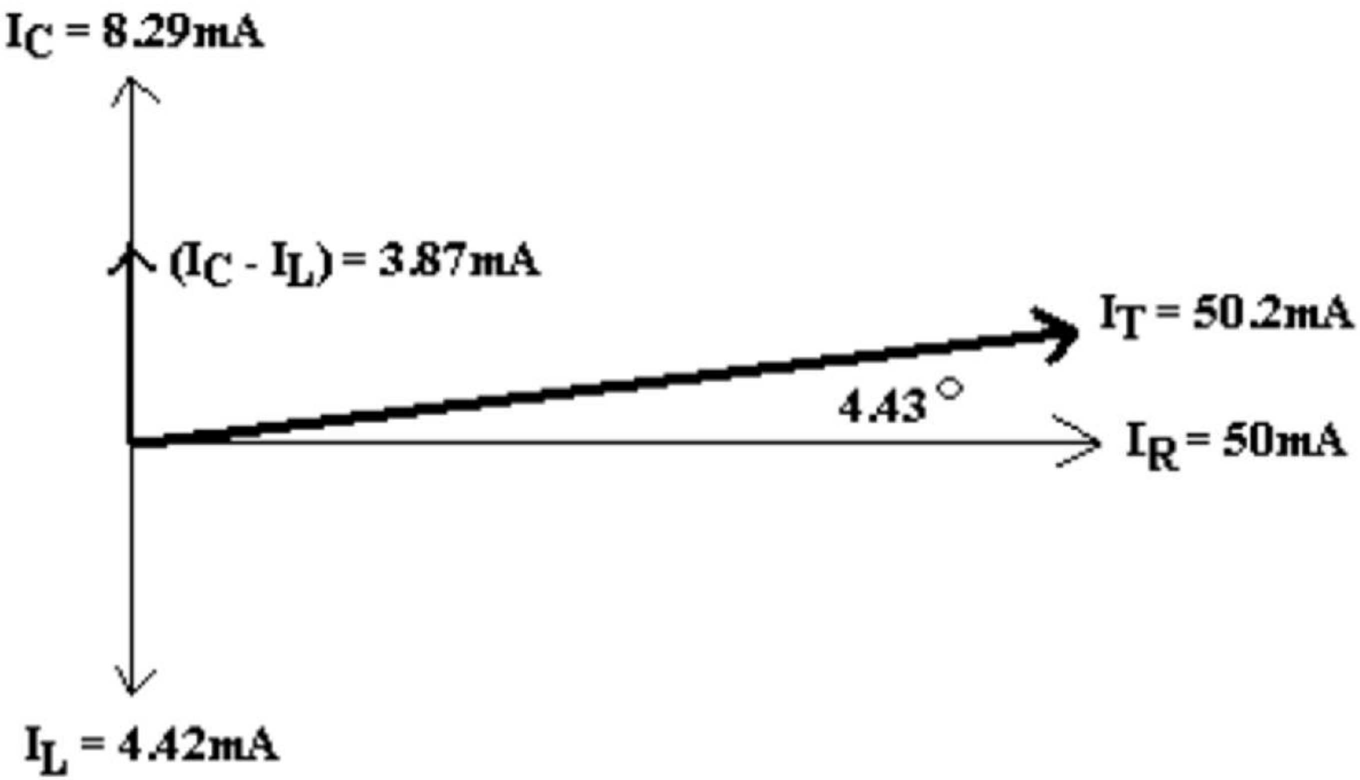
$$I_L = \frac{V_s}{X_L} = \frac{5\angle 0^\circ \text{ V}}{1131\angle 90^\circ \ \Omega} = 4.42\angle -90^\circ \text{ mA}$$

$$I_C = \frac{V_s}{X_C} = \frac{5\angle 0^\circ \text{ V}}{603\angle -90^\circ \ \Omega} = 8.29\angle 90^\circ \text{ mA}$$

$$V_R = V_L = V_C = 5\angle 0^\circ \text{ V}$$

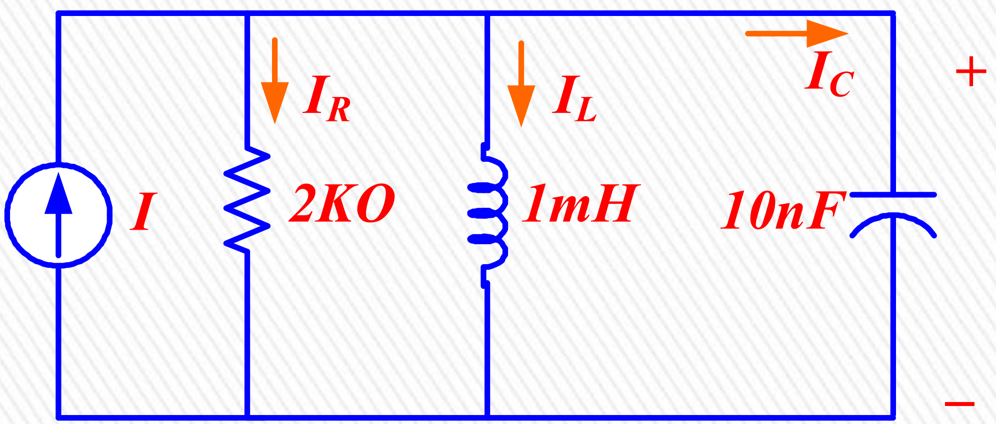
## Example (3)

b). Current phasor diagram:



# Example (4)

Consider the parallel RLC resonant circuit with “ideal” elements given below.




1) The total impedance seen by the source at resonance is

(a) 0 (b) 2KΩ (c) 2MΩ (d) infinity

Ans: (b)

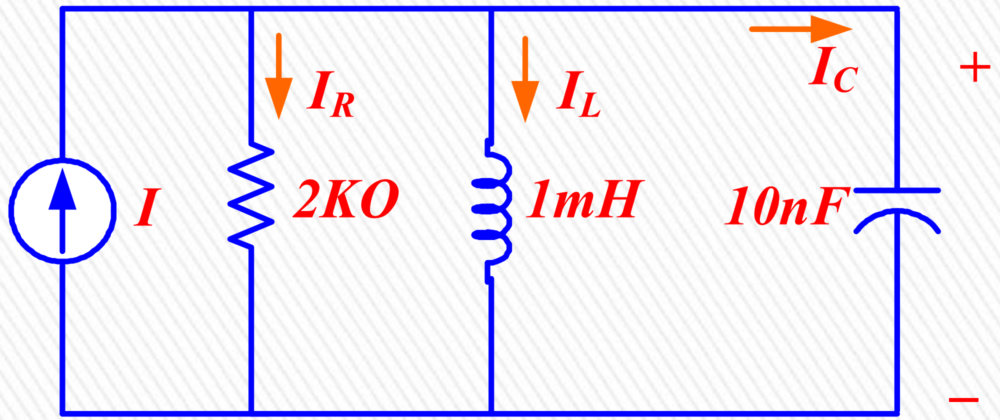
2) The resonant frequency approximately is

(a) 10 Hz (b) 100 KHz (c) 160 KHz (d) 500 KHz

Ans: (c)

# SELF-TEST

Consider the parallel RLC resonant circuit with "ideal" elements given below.




2) The current through the resistance ($I_R$) at resonance is

(a) 0 (b) 1 mA (c) 2 mA (d) 10 mA

Ans: (c)

3) 4) The inductor current $I_L$ at resonance is

a) Equal to $I_R$
b) Equal to $I_C$
c) Equal and opposite to $I_C$
d) Equal to I

Ans: (c)

# Thank You